Problem Set 2 - Partisan politics

Political Economics II (EC38011) Spring 2022

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Instructions

- It is optional, but highly encouraged to submit solutions to the problem sets.
- As discussed please do work on either problem 3 or 4 from PS 1 if you have not submitted anything on them (or if you want to update your answers).
- I also want to acknowledge that the paper in problem 3 is not on the reading list - so it you should skip one problem - it might be that one.
- In order to submit either email them to mattias.folkestad@iies.su.se please submit a zip-file if you have multiple document, such as code.
- If you want feedback do hand them in a day or so before the TA-session on March 1st.

Problems

1. **The citizen-candidate model:** Problem 5.7.2 in Persson et al. (2000)

Solution: Before providing a solution, lets try to iron out some of the confusion with strategic and sincere voting and other "hidden" model features. (For an extensive discussion see Besley and Coate (1997)).

First lets define an equilibrium as:

1. a vector of entry decisions that is an equilibrium in the entry stage.
2. a function of voting behaviour that is an equilibrium in the voting stage.

With three caveats, first we only consider pure entry strategies, that is either run or not run. Mixed strategies are however conceptually possible. Second for the voting stage we require that all equilibrium voting strategies are not weakly dominated. Third if there is only one candidate she wins for all voting strategies.

A voting function is a mapping for the set of candidates running in the election to a voting decision. With strategic voting we mean that voting is decided as an argmax of the expected utility from voting for a candidate given the voting function of everyone else. Sincere voting is when everyone just vote for the candidate who will implement the policy that maximizes the citizens utility.

The requirement that equilibrium voting strategies cannot be weakly dominated implies that you will get sincere voting in any two candidate equilibrium, even though citizens are strategically motivated. For example in a two candidate election, everyone voting for one candidate is not an equilibrium as there are other weakly undominated strategies.

a) Politicians have the same preferences as citizens, and once they win, they just implement their bliss point tax rate τ^i.

$$\tau^i = argmax_\tau \sqrt{(1-\tau)y^i} + \sqrt{\tau y}$$

Note that average income, $y = 1$ The solution is thus: $\tau^i = \frac{1}{y^i+1}$

b) If the median citizen runs, she is the Condorcet winner, and thus it will not be a best response for anyone else to run. The probability of winning is zero due to sincere voting.

Thus the median citizen one candidate equilibrium only require that the participation constraint is satisfied.

$$w^m(\tau^m) - w^m(\bar{\tau}) > \epsilon$$

Note that $w^m(\tau) == \sqrt{(1-\tau)} + \sqrt{\tau}$.

The analytical solution is that $\tau < \frac{1}{4}$ or $\tau > \frac{3}{4}$.

Are there other one candidate equilibrium? Yes! For a another candidate K, the same participation constraint have to hold (so K cannot be to dissimilar to M).

But in addition we have to make sure that the candidate is "immune" to challenges. W.l.o.g. assume $y^K < y^M$, then any challenger C with $y^C < y^K$ will have no chance of winning. The same is true if $y^C > 2y^M - y^K$ but for $y^C = 2y^M - y^K$ we have a 50/50 vote outcome and thus we must have that:

$$w^C(\tau^C) - w^C(\tau^K) < 2\epsilon$$

And for $y^C \in (y^K, 2y^M - y^K)$:

$$w^C(\tau^C) - w^C(\tau^K) < \epsilon$$

Note that the second condition implies the first for our case with sufficiently large ϵ. Will this ever hold? Yes, clearly. One can numerically verify it for some $\bar{\tau}$.

c) The first condition is that there must be a positive probability for both candidates to win. In this setting it is equivalent to saying that the median voter is indifferent between the two candidates.

There is also the same kind of participation constraint as in b). Together we have that for candidates $P \in \{A, B\}$ with $^{A} \neq y^{B}$:

$$w^m(\tau^A) = w^m(\tau^B) \implies \tau^A = (1 - \tau^B)$$
$$w^P(\tau^P) - w^P(\tau^{P'}) > 2\epsilon$$

The voting strategies that allows for this equilibrium are: If two candidates - vote sincerely, if three candidates, vote for the same candidate you would have if only two of them ran.

So why is this an equilibrium? Consider first the voting stage. No citizen can improve by changing the voting strategy. Everyone is potentially pivotal, if one changes their vote they will hand the election to their less preferred candidate.

But what about a voting strategy that is something like: vote for the new candidate if sufficiently many will do it for them to win. This is possible - however this is not an equilibrium, because then the entry game equilibrium breaks down, A and B is not going to pay ϵ to participate unless they have a chance of winning.

But is there not an equilibrium where a third candidate enter so that votes are divided in equal thirds - thus preserving a positive probability of winning. There might be! But as discussed in class, these models typically have multiple equilibria. But first just note that the participation constraints are quite different in the three candidate scenario.

d) See c)

2. **Political rents with endogenous value of being in office:** Problem 5.7.6 in Persson et al. (2000)

Solution:

a) The important insight in this problem is that at the last stage the proposer will just take all the rent. By just giving ϵ to a majority of her colleagues, the proposer can make sure she get a majority for the proposition. Thus the continuation value for any (non proposing) legislator in the first round is just $\beta r \frac{1}{n}$ since there is a $1/n$ probability of being the proposer in the second round, and the value of not proposing in the second is just zero.

Then it is straight forward to see that the proposer will choose $\frac{n-1}{2}$ other members and offer them their continuation value of exactly $\beta r \frac{1}{n}$. Then the proposer gets:

$$v = r - \beta r \frac{1}{n} \frac{n-1}{2} = r\left(1 - \beta\frac{1}{2} + \beta\frac{1}{2n}\right)$$

Will it ever be beneficial to *gamble* on the possibility to become the proposer once again, and then be able to take the whole rent? The answer is no. Formally for $\beta \in [0, 1]$ the inequality:

$$r\left(1 - \frac{n-1}{2n}\beta\right) > \beta r \frac{1}{n}$$

always hold.

Now we can define the advantage of being the first proposer as the difference in expected value from being the proposer and not being the proposer. But what is the expected value of not being the proposer?

Lets say that the proposer randomly chooses who's votes to *buy*. Then with probability $1/2$ you are part of the winning coalition, and there is no chance of becoming the proposer, since the bargaining never goes to the second round. Thus the value of being the first proposer is:

$$\begin{aligned}
\alpha &= v - \frac{1}{2}\beta r \frac{1}{n} \frac{n-1}{2} \\
&= r\left(1 - \beta\left(\frac{n-1}{2n} + \frac{n-1}{4n}\right)\right) \\
&= r\left(1 - \beta\frac{3}{4} + \beta\frac{3}{4n}\right)
\end{aligned}$$

First of all we note that both the rents the proposer gets and the benefit from being the proposer is decreasing in the number of legislators. Note in particular that as $n \to \infty$ $v \to r(1 - \beta/2)$ and $\alpha \to r(1 - 3\beta/4)$

Since the highest β we can have is unity. The *worst case* for the proposer is to get half of the rents.

A, perhaps counter intuitive, result is that both the rents and the benefit is also decreasing in β so the very impatient proposer (low β) gets high rents.

How is that possible - well it is because the β is the same for all legislators.

So what happens it β^i instead follows a distribution? Well if it is common knowledge, then the smallest winning coalition always consists of the $(n-1)/2$ legislators with the lowest β.

What if β^i is individual knowledge? Well, if they can not communicate before the vote then lets say that at least the distribution of β^i are known. Then the objective for the first proposer is: $\max_v vp(v) + (1 - p(v))\beta^P r \frac{1}{n}$ where $p(v)$ is the probability of winning the vote, which depends on the distribution of β in the obvious way.

What if legislators can signal their β^i, if there is no cost to signaling, then all legislators signal a low β to be a part of the winning coalition. Thus the signal is not informative, i.e. cheep talk.

If signaling is costly, you will potentially get separating equilibrium etc...

b) In this setting the continuation value for a non proposing legislator consists of five parts:

$$w^i = \underbrace{\frac{1}{n}}_{\mathbb{P}(\text{proposing})} \underbrace{r\left[1 - \frac{n-1}{2}\beta w^i\right]}_{\text{rents from forming coalition}} + \underbrace{\frac{n-1}{n}}_{\mathbb{P}(\text{not proposing})} \underbrace{\frac{1}{2}}_{\mathbb{P}(\text{part of wining coalition})} \underbrace{\beta r w^i}_{\text{value of wining coalition}}$$

Solving for w^i give $w^i = \frac{r}{n}$. So the proposer have to offer at least $\beta\frac{r}{n}$ for a legislator to vote yes.

3. **Politician characteristic regression discontinuity:** In a recent paper Marshall (2022) critiques the literature using close election RD design to estimate the effect of politicians characteristics on down stream outcomes. Choose a paper on the reading list that uses this methodology and discuss the findings in the light of Marshall's critique.

Solution:

References

Besley, T. and S. Coate (1997). An economic model of representative democracy. *The Quarterly Journal of Economics 112*(1), 85–114.

Marshall, J. (2022). Can Close Election Regression Discontinuity Designs Identify Effects of Winning Politician Characteristics? *American Journal of Political Science*.

Persson, T., G. Tabellini, et al. (2000). Political economics.