Problem Set 1 - Electoral competition and voter behaviour

Political Economics II (EC38011) Spring 2024

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Instructions

It is optional, but highly encouraged to submit solutions to the problem sets. My suggestion for problem 3 and 4 is to choose one and do a through job with the empirical part. I think the learning will be higher from doing one well rather than both sloppy.

In order to submit either email them to mattias.folkestad@iies.su.se please submit a zip-file if you have multiple document, such as code.

If you want feedback do hand them in a day or so before the TA-session on February 9th.

Problems

1. **Noncrediable comittments and probabalistic voting** Problem 3.8.1 in Persson et al. (2000)

Solution: In case someone is paying close attention to details, it is reasonable to make the additional assumption that $\alpha^P \in (0, 1)$

a) The chosen policy is given by the maximization problem:

$$q_2^{P*} = \arg\max_{q_2} \ln(y - q_1 - q_2) + \alpha^P \ln(q_1) + (1 - \alpha^P) \ln(q_2)$$

The policy will satisfy the FOC:

$$\frac{1}{y - q_1 - q_2} = \frac{1 - \alpha^P}{q_2} \iff q_2 = \frac{1 - \alpha^P}{2 - \alpha}(y - q_1)$$

And to verify a maximum either take SOC, or just notice that the objective function is strictly concave in q_2

This can be thought of as function or the winning politicians type and the campaign platform: $q_2^P(q_1^P, \alpha^P)$.

So for a voter the expected utility of a platform from politician P is:

$$
\begin{aligned}
&\mathbb{E}_{F^P}[\ln\{y - q_1^P - q_2(q_1^P, \alpha)\} + \alpha^i \ln(q_1^P) + (1-\alpha^i)\ln\{q_2(q_1^P, \alpha)\}] \\
&= \int_{(0,1)} \left[\ln(y - q_1^P - \frac{1-\alpha}{2-\alpha}(y - q_1^P)) + \alpha^i \ln(q_1^P) + (1-\alpha^i)\ln(\frac{1-\alpha}{2-\alpha}(y - q_1^P)) \right] dF^P(\alpha) \\
&= \int_{(0,1)} \left[\ln\left(\frac{y - q_1^P}{2-\alpha}\right) + \alpha^i \ln(q_1^P) + (1-\alpha^i)\ln\left(\frac{1-\alpha}{2-\alpha}(y - q_1^P)\right) \right] dF^P(\alpha) \\
&= W(q_1^P, \alpha^i, F^P)
\end{aligned}
$$

The preferred policy is thus: $\arg\max_{q_1^P} W(q_1^P, \alpha^i, F^P)$ which clearly depends on the beliefs.

b) Votes are characterized by α^i so a swing voter has α^s s.t. $W(q_1^B, \alpha^s) = W(q_1^A, \alpha^s)$. So that answers the first question.

To find the vote share of the two parties we need to find the share of α^i's where the function $W(q_1^B, \alpha^i) - W(q_1^A, \alpha^i)$ is positive/negative since that determines the voting decision. The argument is somewhat tedious and as discussed in class perhaps not so intuitive.

By starting from the back, what we want is a function that maps a policy pairs to vote shares $(q_1^A, q_1^B) \to (\pi_A, \pi_B)$. As said, a policy pair implicitly defines a swing voter $\alpha^s(q_1^A, q_1^B)$. So why don't we just say that $\pi_P = F(\alpha^s(q_1^A, q_1^B)$? Two reasons: First it does not give the vote share for party A and B, it is the vote share for a party, but not which party. Second we need to show that the swing voter actually split the electorate in two halves where one vote for party A and the other for party B (i.e. we need to verify that preferences satisfy the single-crossing property).

So what to do? Lets think more about the function $\mathcal{W}(q_1^A, q_1^B, \alpha^i) = W(q_1^B, \alpha^i) - W(q_1^A, \alpha^i)$, the expected payoff if B win. Following out desire to find a swing voter that split the electorate we need to differentiate this function w.r.t. α^i. It will tell us how the payoff changes with the type for a given policy pair. That the indirect utility function is linear, which will come in handy.

$$
\begin{aligned}
\frac{\partial \mathcal{W}}{\partial \alpha^i} &= \phi(F^B, F^A, q_1^A, q_2^B) \\
&= \ln(q_1^B) - \ln(q_1^A) - \mathbb{E}_{F^B}\left[\ln\left(\frac{1-\alpha}{2-\alpha}(y - q_1^B)\right)\right] + \mathbb{E}_{F^A}\left[\ln\left(\frac{1-\alpha}{2-\alpha}(y - q_1^A)\right)\right] \\
&= \ln(q_1^B) - \ln(q_1^A) - \ln\left(y - q_1^B\right) - \mathbb{E}_{F^B}\left[\ln\left(\frac{1-\alpha}{2-\alpha}\right)\right] + \ln\left(y - q_1^A\right) + \mathbb{E}_{F^A}\left[\ln\left(\frac{1-\alpha}{2-\alpha}\right)\right] \\
&= \ln\left(\frac{q_1^B}{y - q_1^B}\right) - \ln\left(\frac{q_1^A}{y - q_1^A}\right) + C
\end{aligned}
$$

So what can we say about this function that helps us? Well recall that a feasible $q_1 \in (0, y)$ So we can evaluate the limits of ϕ. Thus first fix the policy q_1^A and then note that the limits w.r.t. q_1^B are $\pm\infty$. So the derivative must have a root (for fixed q_1^A). Is it only one root? Well it clear that $\ln\left(\frac{q_1^B}{y-q_1^B}\right)$ is increasing, thus only one root. (Partially differentiate ϕ w.r.t. q_1^B for a formal argument).

Now we are making progress! There exists a unique q_1^B for every q_1^A such that $\phi(\cdot) = 0$ and at that policy the voters decision rule $\mathcal{W}$ will not depend on voter types, i.e. all voters vote the same way. For what party depends on the value of $\mathcal{W}(\cdot)$ evaluated at that policy. For positive values 100 percent vote for B for negative 100 percent vote for A and when it is zero all voters are indifferent and vote by the flip of a coin thus A and B get 50 percent each.

But this is clearly not the full story. It just says that for each policy of party A there exists a policy for party B such that vote shares are as described. But what about all others cases?

Now define the function $q_1^B(q_1^A)$ as this unique policy that give $\phi(\cdot) = 0$. For all policies below i.e. $q_1^B < q_1^B(q_1^A)$ $\phi(\cdot) < 0$ and vice versa.

The interpretation is that there is a monotonic (positive/negative) relationship between α^i and the $\mathcal{W}(\cdot)$ function for that part in the policy space. We have now verified single crossing!

So the vote share for the whole policy space given by the distribution $F(\cdot)$.

$$\pi_B = \begin{cases} F(\alpha^s(F^B, F^A, q_1^A, q_2^B)) & q_1^B < q_1^B(q_1^A) \\ \{1, 1/2, 0\} & q_1^B = q_1^B(q_1^A) \\ 1 - F(\alpha^s(F^B, F^A, q_1^A, q_2^B)) & q_1^B > q_1^B(q_1^A) \end{cases}$$

And as always $\pi_A = 1 - \pi_B$.

c) If beliefs are the same we have $C = 0$. Then it is clear that the "split-the-vote"-response $q_1^B(q_1^A) = q_1^A$ i.e. policy convergence in equilibrium.

When they are different the parties clearly have to run on different platforms to split the vote.

d) (Extra) Note that the question does not explicitly ask about equilibrium. It is clear however that in the case with similar beliefs that policy converges to the median voters bliss point q_1^m. The Nash equilibrium strategy for party P is:

$$q_1^P = \begin{cases} q_1^m & \text{for} \quad q_1^{P'} = q_1^m \\ q_1^P > q_1^{P'} & \text{for} \quad q_1^{P'} < q_1^m \\ q_1^P < q_1^{P'} & \text{for} \quad q_1^{P'} > q_1^m \end{cases}$$

The argument is similar for different beliefs, but policy will not be the same. So in welfare terms we can compare the two scenarios by taking the expected welfare and compare it to the median voter equilibrium.

I also made another observation - I mentioned in class that politicians are office motivated in this model - well, they will act that way at least, since they will always benefit from choosing q_2. Except from the following scenario.

Lets assume politicians type are the same $\alpha^A = \alpha^B$ but that voters beliefs are quite different. And add the critical assumption that politicians know the type of the other. Then another equilibrium exists where one party drop out of the race (or suggest a policy that they know will loose) and let the winning party campaign on the q_1 that represent the politicians joint bliss point.

Formally one should think of this as a participation constraint for both parties. If the utility for the other party running uncontested (i.e. choose q_1, q_2 freely) is greater than the expected utility in equilibrium then they will not run.

Perhaps not empirically relevant (in particular if we think och preference for q_2 as ideology) but it highlights two important aspect of modeling political competition. Time order and information!

2. **Lobbying** Problem 3.8.5 in Persson et al. (2000)

Solution:

a) Voters who are indifferent between two candidates (i.e. swing voters) are described by

$$W(q^A;\alpha^J) = W(q^B;\alpha^J) + h \cdot (C_B - C_A) + \sigma^J + \tilde{\delta} \quad \text{for each } J,$$

where $C_P := \sum_J O^J \lambda^J C_P^J$ for $P \in \{A, B\}$.

Candidate A's vote share is given by

$$\pi_A = \sum_J \lambda^J \phi^J \left(\sigma^J + \frac{1}{2\phi^J}\right),$$

and the candidate's probability of winning is written by

$$p_A = \frac{1}{2} + \frac{\psi}{\phi} \sum_J \lambda^J \phi^J (W(q^A;\alpha^J) - W(q^B;\alpha^J)) + \psi h \cdot (C_A - C_B).$$

b) The objective function of each member in group J is given by

$$\mathcal{U}^J = \underbrace{p_A W(q^A;\alpha^J) + (1-p_A) W(q^B;\alpha^J)}_{\text{Expected utility for the election}} - \underbrace{\frac{1}{2}(C_A^J + C_B^J)^2}_{\text{Cost of contributions}}.$$

Each group member maximizes the objective function to determine the amount of group contribution for each party. Since you can not make negative contributions, only positive contributions to the opponent, the maximization require some KT-conditions. Formally:

$$\max \mathcal{U}^J(\cdot) \quad wrt \quad C_A^J, C_B^J \quad st \quad C_A^J, C_B^J \geq 0 \tag{1}$$

This problem have the lagrangian:

$$\mathcal{L}^J = \mathcal{U}^J(\cdot) + \mu_A C_A^J + \mu_B C_B^J$$

And thus KT-conditions

$$\frac{\partial \mathcal{U}^J}{\partial C_A^J} + \mu_A = 0$$
$$\frac{\partial \mathcal{U}^J}{\partial C_B^J} + \mu_B = 0$$
$$\mu_A \geq 0 \text{ , with } \mu_A = 0 \text{ if } C_A > 0$$
$$\mu_B \geq 0 \text{ , with } \mu_B = 0 \text{ if } C_B > 0$$

From here is should be clear that the sign on the partial derivatives depends on the sign of $W_A^J - W_B^J$:

$$\frac{\partial \mathcal{U}}{\partial C_A^J} = \frac{\partial p_A}{\partial C_A^J}(W_A^J - W_B^J) - (C_A^J + C_B^J) = \psi h \lambda^J (W_A^J - W_B^J) - (C_A^J + C_B^J)$$
$$\frac{\partial \mathcal{U}}{\partial C_B^J} = \frac{\partial p_A}{\partial C_B^J}(W_A^J - W_B^J) - (C_A^J + C_B^J) = -\psi h \lambda^J (W_A^J - W_B^J) - (C_A^J + C_B^J)$$

First note that when $W_A^J - W_B^J = 0$ then the optimal contribution is zero to both parties.

Next consider the case when $W_A^J - W_B^J > 0$ then zero contribution is not optimal as $\frac{\partial \mathcal{U}^J}{\partial C_A^J} > 0$ so $C_A^J > 0$ implying that $\mu_A = 0$ and thus $\mu_B > 0$ i.e. that condition binds and $C_B^J = 0$. The optimal level of contribution to party A is then just $C_A^J = \psi h \lambda^J (W_A^J - W_B^J)$. The argument is similar for party B.

This whole thing can be summarized as:

$$C_P^{J*} = \max\{0, \psi h \lambda^J (W_P^J - W_{P'}^J)\}$$

where P' denotes the other candidate.

c) Given C_P^{J*}, candidate A determines a platform by maximizing own probability of winning:

$$p_A = \frac{1}{2} + \frac{\psi}{\phi} \sum_J \lambda^J \phi^J (W(q^A; \alpha^J) - W(q^B; \alpha^J)) + \psi h (C_A^* - C_B^*),$$

where $C_A^* := \sum_J O^J \lambda^J C_A^{J*} = \psi h \sum_J O^J (\lambda^J)^2 (W_A^J - W_B^J)$ and $C_B^* := 0$. This is equivalent to maximize

$$\sum_J \lambda^J \left(\frac{\phi^J}{\phi} + \psi h^2 O^J \lambda^J \right) W_A^J,$$

with respect to q, which yields

$$q^{lob} := V_q^{-1}\left(\frac{\sum_J \lambda^J\left(\frac{\phi^J}{\phi}+\psi h^2 O^J\lambda^J\right)}{\sum_J \lambda^J\left(\frac{\phi^J}{\phi}+\psi h^2 O^J\lambda^J\right)\alpha^J}\right).$$

Let $O^J = 0$ for all J. Then $q^{lob} = q^g$. In other words, if no groups finance the candidates, the model boils down to the standard problem with groups of voters (Problem 3.8.4 in Persson et al. (2000)), and we get the same equilibrium. Assuming $\phi^J = \phi$ for all J makes the equilibrium platform be socially optimal.

Next, let $O^J = 1$ for all J. Then we have

$$q^{lob} := V_q^{-1}\left(\frac{\sum_J \lambda^J\left(\frac{\phi^J}{\phi}+\psi h^2\lambda^J\right)}{\sum_J \lambda^J\left(\frac{\phi^J}{\phi}+\psi h^2\lambda^J\right)\alpha^J}\right).$$

Assuming $\phi^J = \phi$ for all J yields

$$q^{lob} := V_q^{-1}\left(\frac{\sum_J \lambda^J\left(1+\psi h^2\lambda^J\right)}{\sum_J \lambda^J\left(1+\psi h^2\lambda^J\right)\alpha^J}\right).$$

Finally, assuming $\lambda^J = \lambda$ for all J yields $q^{lob} = q^*$. In other words, if we assume that all groups have the same size ($\lambda^J = \lambda$) in addition to the same density assumption ($\phi^J = \phi$), the equilibrium coincides with the social optimum, because candidates do not put weight ϕ^J on each group, and citizen groups do not put weight λ^J on their contribution.

d) Let $J \in \mathcal{J}' \subset \mathcal{J}$ where $\mathcal{J}$ is a set of all groups and $\mathcal{J}'$ is a set of groups which contribute. Then the expression (c)) is re-written by

$$q^{lob} := V_q^{-1}\left(\frac{1+\sum_{J\in\mathcal{J}'}\psi h^2\lambda^J}{\frac{1}{\phi}\sum_{\mathcal{J}}\lambda^J\phi^J\alpha^J+\sum_{J\in\mathcal{J}'}\psi h^2\lambda^J\alpha^J}\right).$$

The equilibrium platform is tilted in favor of organized groups' policy preferences. Moreover, the candidates also consider the size of organized groups (λ). People who take large stake in economy may have a high incentive to form a group, although too big group may face free-riding problems.

3. **Women's suffrage.** The role of women voters in the expansion of the government in the US is the topic of Lott and Kenny (1999). In this exercise we will see how the relationship hold up if we include more countries in the sample. We also learned that voter turnout keep on increasing after a reform that expanded the franchise, in this particular case female suffrage in the US. In general newly franchised groups have an initially lower turnout (see Morgan-Collins (2023) for deeper analysis of the case of women.).

   a) Use the data on government expenditure and year of female suffrage provided (or by all means find alternative sources) in order to analyse the relationship between female suffrage expansion and the size of government. Discuss your findings.

b) Discuss some of the plausible explanations for the initial difference in turnout between men and women after female enfranchisement.

c) Are there any reasons to ex-ante expect convergence in turnout? For gender gap in particular and other suffrage extensions in general?

d) Discuss some of the factors that should correlate with faster/slower/any closing of the turnout gap.

e) (Extra) Find some data on the gender (or other) turnout gap for other countries than the US? Can some of the hypothesis discussed be tested?

4. **Moral values and voting.** In this exercise we are revisiting Enke (2020). Enke provides a model for probabilistic voting and extends the model with moral values with interesting predictions.

   a) Using the conceptual framework presented in section II of the paper derive the optimal level of moral universalism for a presidential candidate. I.e the parameter θ_j. For a closed form solution we need a distributional assumption on the popularity shock ϵ which can be set to uniform with density ϕ. To further simplify assume that voters are homogeneous in nonmoral characteristics ($x_i = x$).

   b) Use the results from a) to discuss if there are empirical support for your results and why/why not they would hold up in the real world.

   c) By using the replication files and data provided for the US 2020 presidential election you will now extend Enke's analysis in table 6 with another elections year. Discuss your results and the implications for external validity of the findings in the paper.

   d) (Extra) If you are interested I have also added text-data for campaign rhetoric for Donald Trump and Joe Biden in the 2020 election. Using the raw data and following the methodology outlined in the paper one can reproduce figure 6A with another elections year added which perhaps is an even better test for the usefulness of the model.

Solution:

a) In order to simplify notation lets just use R, D for parties and r, d for candidates. Note that the subscripts on α, β thus become redundant. The expected vote share for the Democrats are:

$$\pi_D = \frac{1}{\mathbb{I}} \sum_{i=1}^{\mathbf{I}} \pi_D^i$$

We get an expressing for π_D^i in equation (5) and with our assumption about homogeneity in nonmoral characteristics we can get the simpler version (We can just skip the policy part since candidates are office oriented they will choose the same policy.):

$$\pi_D^i = \mathbb{P}(\alpha + \beta\theta_i > \epsilon_i)$$

Now using the distributional assumption on the idiosyncratic popularity shock we get:

$$\begin{aligned}\pi_D^i &= \mathbb{P}(\epsilon_i \leq -(\alpha + \beta\theta_i)) \\ &= \phi(\frac{1}{2\phi} - (\alpha + \beta\theta_i)) \\ &= \frac{1}{2} - \phi(\alpha + \beta\theta_i)\end{aligned}$$

Thus $\pi_D = \frac{1}{2} - \phi(\alpha + \beta\overline{\theta})$ where $\overline{\theta}$ is the average level or universalist moral values in the electorate. Note now that this is the expected vote share, and we will assume that the candidate maximizes wrt this objective, rather than winning probabilities which is usually unproblematic (see page 34 in Persson et al. (2000)).

The choice variables here is the candidates own moral stance θ_j. For Democratic candidate this give FOC:

$$\begin{aligned}&\frac{\partial \pi_D}{\partial \theta_d} = 0 \\ \implies & -\phi(\frac{\partial \alpha}{\partial \theta_d} + \frac{\partial \beta}{\partial \theta_d}\overline{\theta})) = 0 \\ \implies & \frac{\partial(-\lambda(\gamma^2\theta_d^2 + 2\gamma(1-\gamma)\theta_d\theta_D))}{\partial \theta_d} + 2\lambda\gamma\overline{\theta} = 0 \\ \implies & -\lambda(2\gamma^2\theta_d^* + 2\gamma(1-\gamma)\theta_D) + 2\lambda\gamma\overline{\theta} = 0 \\ \implies & \theta_d^* = \frac{1}{\gamma}(\overline{\theta} - \theta_D) + \theta_D\end{aligned}$$

The problem is symmetrical for the republican candidate.

Not surprisingly the candidate want to compensate for the parties deviations from the average moral values in the electorate. One perhaps interesting implication is the role of the γ-parameter. When γ is close to unity candidates form both parties will just signal the same moral values as the average voter. But if we have a low γ i.e. voters think party identity is more important than candidate characteristics candidates will have to become more extreme in their deviations from the party average in order to attract voters.

b) Given the empirical results in the paper the democratic party has higher universalist values $\theta_D > \theta_R$. This also implies that optimal strategy for the democratic candidate always is less universalist than the republican candidate (in the general election that is). Unfortunately the evidence presented in the paper cannot really speak to this. The measures of the political rhetoric in for individual candidates (fig 3) are taken from the primaries, not the general election campaign.

However it is interesting to note that in the three elections studied only 2008 with this prediction. Obama is scored as less universalist then McCain. As we will see in d) this is also the case in 2020.

Is this a surprise? Well nothing in the world suggest that candidates are chosen optimally. But perhaps they should communicate optimally once they start the general election campaign?

c) See table 1. Code in separate file.

d) See figure 1. Code in separate file.

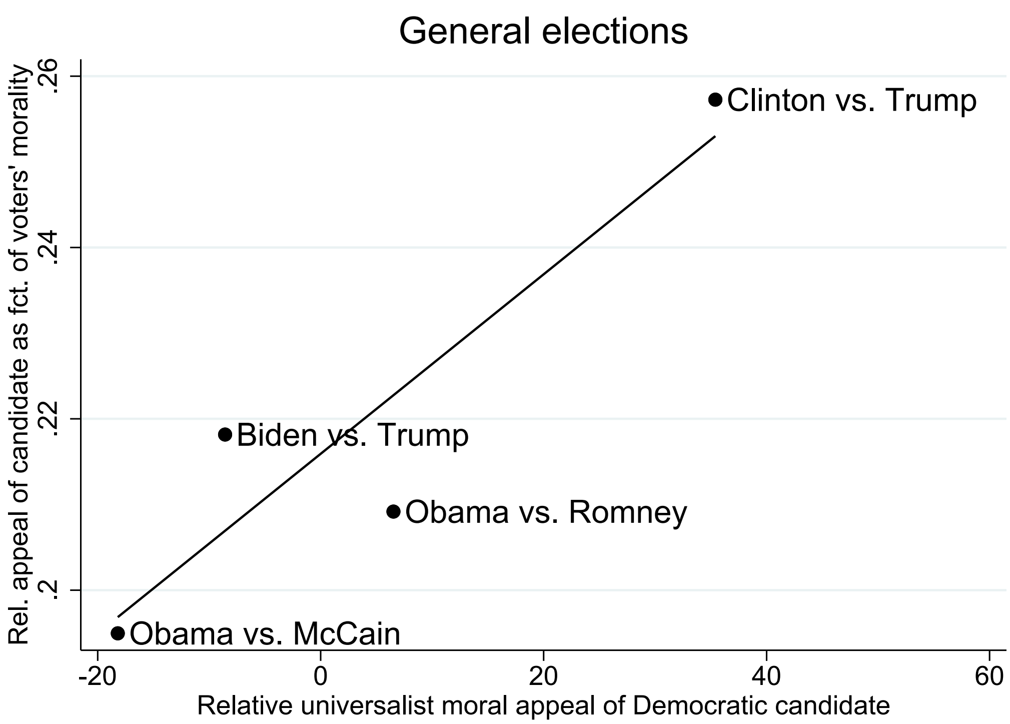

Figure 1: Figure 6A, Enke 2021

References

Enke, B. (2020, October). Moral Values and Voting. *Journal of Political Economy 128*(10), 3679–3729.

Lott, Jr., J. R. and L. W. Kenny (1999, December). Did Women's Suffrage Change the Size and Scope of Government? *Journal of Political Economy 107*(6), 1163–1198.

Morgan-Collins, M. (2023, June). Bringing in the New Votes: Turnout of Women after Enfranchisement. *American Political Science Review*, 1–16.

Persson, T., G. Tabellini, et al. (2000). Political economics.

Table 1: Table 6

	Dependent variable:							
	Vote shares							
	Presidential election							
	Trump				Δ [Trump – Ave. GOP]			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Moral values	-2.38***	-1.64***	-1.41***	-2.02***	-0.58***	-0.32***	-0.23**	-0.18
	(0.30)	(0.25)	(0.24)	(0.57)	(0.13)	(0.10)	(0.10)	(0.27)
Log (HH income)		-2.16	6.46***	4.26		-10.2***	-10.6***	-13.9***
		(1.59)	(1.95)	(3.14)		(0.66)	(0.76)	(1.26)
Unemployment rate		-1.00***	-0.64***	-1.03**		-0.052	0.011	-0.022
		(0.19)	(0.19)	(0.47)		(0.08)	(0.08)	(0.19)
Racism index		1.71***	1.37**	0.016		0.18	-0.067	-0.47
		(0.37)	(0.60)	(2.96)		(0.17)	(0.25)	(0.67)
Log(Population density)		-6.36***	-7.59***	-8.03***		-2.17***	-3.02***	-3.28***
		(0.24)	(0.24)	(0.35)		(0.11)	(0.11)	(0.18)
Fraction religious		11.3***	9.04***	4.39		1.76**	0.66	0.44
		(2.10)	(2.11)	(3.46)		(0.73)	(0.73)	(1.34)
Abs. value of moral values index		-0.16	-0.41	-0.42		0.26	0.034	0.21
		(0.39)	(0.36)	(0.80)		(0.17)	(0.15)	(0.37)
Latitude		0.17	0.27	0.69		-0.045	-0.39	0.29
		(0.21)	(0.71)	(1.38)		(0.09)	(0.28)	(0.63)
Longitude		0.013	-0.013	-0.50		0.19***	0.16	1.03*
		(0.16)	(0.51)	(1.08)		(0.07)	(0.24)	(0.53)
State FE	Yes	Yes	No	No	Yes	Yes	No	No
Commuting zone FE	No	No	Yes	No	No	No	Yes	No
CBSA FE	No	No	No	Yes	No	No	No	Yes
Observations	2256	2214	2214	1629	2256	2214	2214	1629
R^2	0.37	0.62	0.82	0.88	0.31	0.63	0.83	0.87